

K and Matching Logic

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Question

... could it be that, after 40 years of program verification, we still lack the right semantically grounded program verification foundation?

Hoare logic

$\{\pi_{\text{pre}}\} \text{ code } \{\pi_{\text{post}}\}$

Current State-of-the-Art in Program Analysis and Verification

Consider some programming language, L

- **Formal semantics of L ?**
 - Typically skipped: considered expensive and useless
- Model checkers for L
 - Based on some adhoc encodings/models of L
- Program verifiers for L
 - Based on some other adhoc encodings/models of L
- Runtime verifiers for L
 - Based on yet another adhoc encodings/models of L
- ...

Example of C Program

- What should the following program evaluate to?

```
int main(void) {  
    int x = 0;  
    return (x = 1) + (x = 2);  
}
```

- According to the C “standard”, it is **undefined**
- GCC4, MSVC: it returns **4**
GCC3, ICC, Clang: it returns **3**
By April 2011, both Frama-C (with its Jessie verification plugin) and Havoc "prove" it returns **4**

A Formal Semantics Manifesto

- Programming languages must have formal semantics! (period)
 - And analysis/verification tools should build on them at best, or should formally relate to them at worst
 - Otherwise they are adhoc and likely wrong
- Informal manuals are not sufficient
 - Manuals typically have a formal syntax of the language (in an appendix)
 - Why not a formal semantics appendix as well?

Motivation and Goal

- We are facing a semantic chaos
 - Axiomatic, denotational, operational, etc.
- Why so many semantic styles?
 - Since none of them is ideal, they have limitations
- We want a powerful, unified foundation for language design, semantics and verification
 - One semantic approach to serve all the purposes!
 - To work with realistic languages (C, Java, etc.)

Minimal Requirements for an Ideal Language Semantic Framework

- Should be **expressive**
 - Substitution or environment-based definitions, abrupt control changes (callcc), concurrency, etc.
- Should be (efficiently) **executable**
 - So we can test it and use it in tools (symb. exec.)
- Should be **modular** (thus scale)
 - So each feature is defined once and for all
- Should serve as a basis for **program reasoning**
 - So we can also prove programs correct with it

- Conventional Semantic Approaches -
 - Advantages and Limitations –

Chronologically

- 1969: Floyd-Hoare Logic -

- Basis for **program verification**
- Not easily executable, and thus, hard to test
 - Semantic errors found by proving wrong properties
 - Soundness rarely or never proved in practice
- Not very expressive
 - Often requires heavy program transformations (e.g., to eliminate side effects, pointers, exceptions, etc.), to reduce languages to cores which can be given an axiomatic semantics
 - Structural program properties (e.g., about heap, stacks, input/output, etc.) hard to state; need special logic support
 - Structural framing (e.g., heap framing) hard to deal with
- Implementations of Floyd-Hoare verifiers for real languages still an art, who few master

- 1971: Denotational Semantics -

Reasonable trade-offs. “Compiles” programs into mathematical objects, so it can be *in principle*:

- **Expressive**, provided enough/appropriate mathematical domains available
- **Executable**: we can execute/approximate fixed-points
 - Although factorial(5) crashes Papaspyrou’s C semantics
- **Modular**, provided one uses advanced features
 - Monads, continuations, resumptions
- Basis for **program verification**
 - Program = least fixed point, so we can use induction
- Hard to use and understand; requires expert knowledge; no overwhelming evidence it is practical for verification

- 1981: Operational Semantics -

Quite intuitive, easy to understand and define.
Requires minimal training and it scales.

- **Executable**, by its very nature
 - Although Norish's C semantics not executable, evaluation contexts are inefficient, CHAM has no machine support, etc.
- **Expressive** ... in principle
 - Although hard to use both evaluation contexts (for call/CC, longjumps,...) and environment-store (for pointers, threads,...)
- **Modular**, when one uses
 - MSOS ideas for dealing with configuration changes, evaluation contexts ideas for control, CHAM ideas for concurrency, etc.
- Considered “too low level”, inappropriate for verification...

What We Want

- First, we want a semantic framework which, at the same time and uniformly well, is
 - Expressive, Executable, Modular, and
 - Suitable for program reasoning
- Second, we want to develop supporting tools for
 - Defining formal language semantics, and
 - Using the semantics for program verification
 - Put an end to having both operational and axiomatic or other semantics to languages. No more semantics equivalence proofs to be done and maintained!

Our quest

Next I will tell a story about our quest for such a unified and practical semantic framework

Take our approach as a possibility ...
... not as ultimate answer

Message to take home:
This is not a dream!

Starting Point: Rewriting Logic

Meseguer (late 80s, early 90s)

- **Expressive**
 - Any logic can be represented in RL (it is reflective)
- **Executable**
 - Quite efficiently; Maude often outperforms SML
- **Modular**
 - Allows rules to only “match” what they need
- Can serve as a basis for **program reasoning**
 - Admits initial model semantics, so it is amenable for inductive or fixed-point proofs

The K Framework

k-framework.org

- A tool-supported rewrite-based framework for defining programming language semantics
- Main ideas
 - Represent program configurations as potentially **nested structures of cells** (like in the CHAM)
 - Flatten syntax into special **computational structures** (like in refocusing for evaluation contexts)
 - Define the semantics of each language construct by **semantic rules** (a small number, typically 1 or 2)

Complete K Definition of KernelC

```

MODULE KERNELC-SYNTAX
IMPORTS K-LATEX+PL-ID+PL-INT
SYNTAX  $Exp ::= Exp * Exp$  [strict]
      | DeclId
      | Id
      | Int
      |  $Exp \rightarrow Exp$  [strict]
      |  $Exp ++$ 
      |  $Exp == Exp$  [strict]
      |  $Exp != Exp$  [strict]
      |  $Exp < Exp$  [strict]
      |  $Exp \leq Exp$  [strict]
      |  $Exp \% Exp$  [strict]
      | !  $Exp$ 
      |  $Exp \&\& Exp$ 
      |  $Exp ? Exp : Exp$ 
      |  $Exp [] Exp$ 
      | printf("hd",  $Exp$ ) [strict]
      | scanf("hd", & $Exp$ )
      | scanf("hd",  $Exp$ ) [strict]
      | NULL
      | PointerId
      | (int*)malloc(  $Exp$  *sizeof(int)) [strict]
      | free(  $Exp$  ) [strict]
      | *  $Exp$  [strict]
      |  $Exp [] Exp$ 
      |  $Exp = Exp$  [strict(2)]
      | Id ( List(Exp) ) [strict(2)]
      | Id ( )
      | random()
      | srandom(  $Exp$  ) [strict]
SYNTAX Stmt ::= Exp ; [strict]
      | { }
      | { StmtList }
      | if(  $Exp$  ) Stmt
      | if(  $Exp$  ) Stmt else Stmt [strict(1)]
      | while(  $Exp$  ) Stmt
      | return  $Exp$  ; [strict]
      | DeclId List(DeclId) { StmtList }
      | #include< StmtList >
SYNTAX StmtList ::= StmtList StmtList
SYNTAX Pgm ::= StmtList
SYNTAX Id ::= main
SYNTAX PointerId ::= * PointerId [ditto]
      | Id
SYNTAX DeclId ::= int  $Exp$ 
      | void PointerId
SYNTAX StmtList ::= stdio.h
      | stdlib.h
SYNTAX List(Bottom) ::= List(Bottom) , List(Bottom) [assoc hybrid id: ( ) strict]
      | ( )
      | Bottom
SYNTAX List(PointerId) ::= List(PointerId) , List(PointerId) [ditto]
      | List(Bottom)
      | PointerId
SYNTAX List(DeclId) ::= List(DeclId) , List(DeclId) [ditto]
      | DeclId
      | List(Bottom)
SYNTAX List(Exp) ::= List(Exp) , List(Exp) [ditto]
      | Exp
      | List(DeclId)
      | List(PointerId)
END MODULE

```

```

MODULE KERNELC-DESUGARED-SYNTAX
IMPORTS K-LATEX
IMPORTS KERNELC-SYNTAX
MACRO  $I \ E = E ? 0 : 1$ 

MACRO  $E_1 \&\& E_2 = E_1 ? E_2 : 0$ 

MACRO  $E_1 [] E_2 = E_1 ? 1 : E_2$ 

MACRO if(  $E$  )  $S_1 = if( E ) S_1$  else { }

MACRO NULL = 0

MACRO  $I () = I ( () )$ 

MACRO int * PointerId = int PointerId

MACRO #include<  $S_1$  > =  $S_1$ 

MACRO  $E_1 [ E_2 ] = * E_1 + E_2$ 

MACRO scanf("hd", &*  $E$ ) = scanf("hd",  $E$ )

MACRO int * PointerId =  $E = int PointerId = E$ 

MACRO int  $X = E ; = int X ; X = E ;$ 

MACRO stdio.h = { }

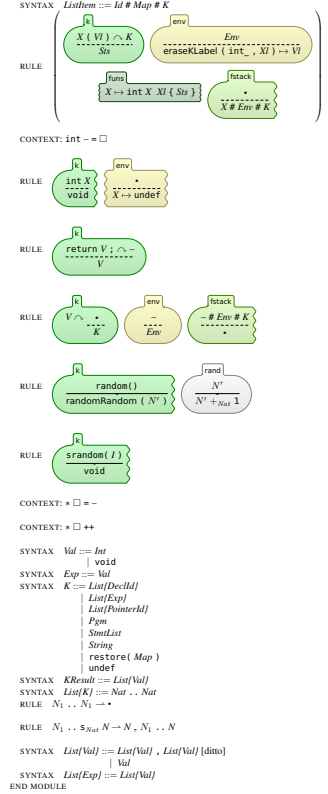
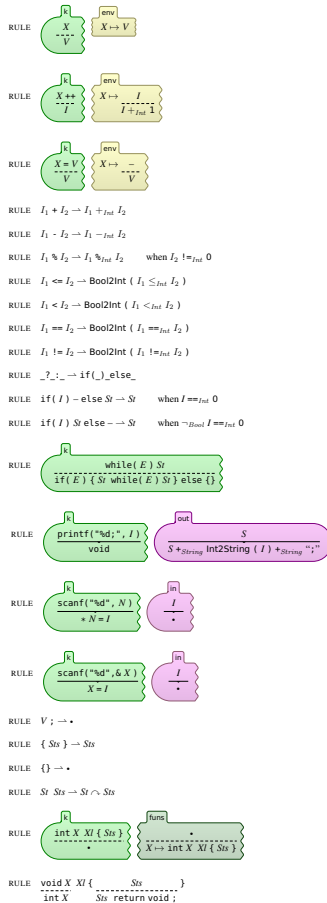
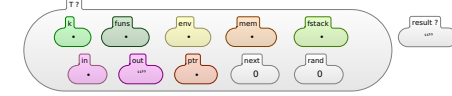
MACRO stdlib.h = { }
END MODULE

```

```

MODULE KERNELC-SEMANTICS
IMPORTS K-SHARED
IMPORTS K-KERNELC-DESUGARED-SYNTAX+PL-CONVERSION+PL-RANDOM
CONFIGURATION:

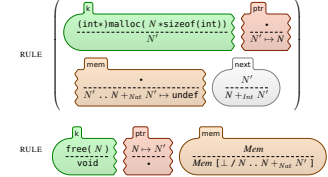
```



```

MODULE KERNELC-SIMPLE-MALLOC
IMPORTS K
IMPORTS KERNELC-SEMANTICS

```



END MODULE

Complete K Definition of KernelC

```

MODULE KERNELC-SYNTAX
IMPORTS K-LATEX+PL-ID+PL-INT
SYNTAX Exp ::= Exp * Exp [strict]
          DeclId
          Id
          int
          Exp > Exp [strict]
          Exp ++
          Exp == Exp [strict]
          Exp != Exp [strict]
          Exp < Exp [strict]
          Exp % Exp [strict]
          ! Exp
          Exp && Exp
          Exp ? Exp : Exp
          Exp || Exp
          printf("hd", Exp) [strict]
          scanf("%hd", &Exp)
          scanf("%hd", Exp) [strict]
          NULL
          PointerId
          (int*)malloc( Exp *sizeof(int)) [strict]
          free( Exp ) [strict]
          * Exp [strict]
          Exp.L Exp.L
          Exp = Exp [strict(2)]
          Id ( List(Exp) ) [strict(2)]
          Id ()
          random()
          srand( Exp ) [strict]
          #include< SmlList >
SYNTAX Stmt ::= Exp ; [strict]
          {}
          { SmlList }
          if( Exp ) Stmt
          if( Exp ) Stmt else Stmt [strict(1)]
          while( Exp ) Stmt
          return Exp ; [strict]
          DeclId List(DeclId) { SmlList }
          #include< SmlList >
SYNTAX SmlList ::= SmlList SmlList
          | Stmt
SYNTAX Pgm ::= SmlList
SYNTAX Id ::= main
SYNTAX PointerId ::= * PointerId [ditto]
          | Id
SYNTAX DeclId ::= int Exp
          | void PointerId
SYNTAX SmlList ::= stdio.h
          | stdlib.h
SYNTAX List(Bottom) ::= List(Bottom) , List(Bottom) [assoc hybrid id: () strict]
          | ()
          | Bottom
SYNTAX List(PointerId) ::= List(PointerId) , List(PointerId) [ditto]
          | List(Bottom)
          | PointerId
SYNTAX List(DeclId) ::= List(DeclId) , List(DeclId) [ditto]
          | DeclId
          | List(Bottom)
SYNTAX List(Exp) ::= List(Exp) , List(Exp) [ditto]
          | Exp
          | List(DeclId)
          | List(PointerId)
END MODULE

MODULE KERNELC-DESUGARED-SYNTAX
IMPORTS K-LATEX
IMPORTS KERNELC-SYNTAX
MACRO ! E = E ? 0 : 1

MACRO E1 && E2 = E1 ? E2 : 0

MACRO E1 || E2 = E1 ? 1 : E2

MACRO if( E ) St = if( E ) St else {}

MACRO NULL = 0

MACRO I () = I ( () )

MACRO int * PointerId = int PointerId

MACRO #include< Smls > = Smls

MACRO E1 [ E2 ] = * E1 * E2

MACRO scanf("%hd", & * E) = scanf("%hd", E)

MACRO int * PointerId = E = int PointerId = E

MACRO int X = E ; = int X ; X = E ;

MACRO stdio.h = {}

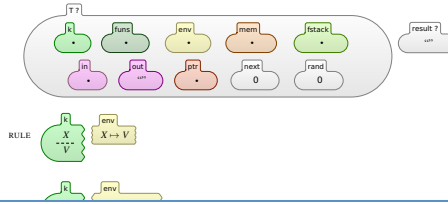
MACRO stdlib.h = {}
END MODULE

```

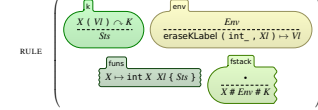
Syntax declared using annotated BNF

SYNTAX $Exp ::=$
 $| Exp = Exp [strict(2)]$

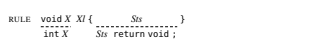
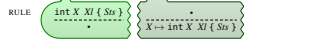
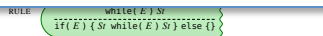
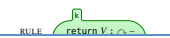
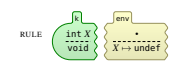
MODULE KERNELC-SEMANTICS
IMPORTS K-SHARED
IMPORTS K-KERNELC-DESUGARED-SYNTAX+PL-CONVERSION+PL-RANDOM
CONFIGURATION:



SYNTAX $ListMem ::= Id \# Map \# K$



CONTEXT: $int_ = \square$



SYNTAX $KResult ::= List(Val)$
SYNTAX $List(K) ::= Nat . Nat$
RULE $N_1 . N_1 ->$

RULE $N_1 . 5_{Nat} N -> N , N_1 . N$

SYNTAX $List(Val) ::= List(Val) , List(Val) [ditto]$
| Val

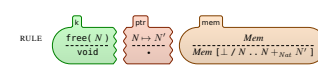
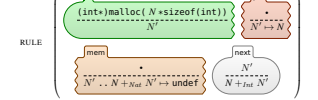
SYNTAX $List(Exp) ::= List(Val)$

END MODULE

MODULE KERNELC-SIMPLE-MALLOC

IMPORTS K

IMPORTS KERNELC-SEMANTICS



END MODULE

```

MODULE KERNEL-SYNTAX
IMPORTS K-LATEX-PL-ID+PL-INT
SYNTAX  Exp ::= Exp + Exp [strict]
          | Declid
          | Id
          | Int
          | Exp - Exp [strict]
          | Exp ++
          | Exp == Exp [strict]
          | Exp != Exp [strict]
          | Exp <= Exp [strict]
          | Exp < Exp [strict]
          | Exp % Exp [strict]
          | ! Exp
          | Exp && Exp
          | Exp ? Exp : Exp
          | Exp || Exp
          | print("vd", , Exp) [strict]
          | scanf("%d", &Exp) [strict]
          | scanf("%d", , Exp) [strict]
          | NULL
          | PointerId
          | (int*)
          | if

```

[illegible]

The diagram illustrates a stack frame structure. It consists of a large rounded rectangle containing ten smaller, colored, pill-shaped blocks arranged in two rows. The top row has five blocks: 'k' (green), 'funcs' (light green), 'env' (yellow), 'mem' (orange), and 'fstack' (light green). The bottom row has five blocks: 'in' (pink), 'out' (purple), 'ptr' (orange), 'next' (light gray), and 'rand' (light gray). Each block has a small black dot in the center. Above the 'k' block is a small tab labeled 'T'.

RULE $\frac{\text{void } X \text{ } XI \{ \quad \quad \quad S_{ts} \quad \quad \quad }{\text{int } X \quad \quad \quad S_{ts} \text{ return void ;}}$

END MODULE

Complete K Definition of KernelC

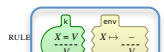
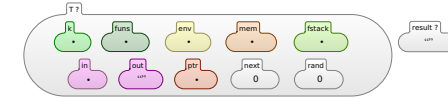
```

MODULE KERNELC-SYNTAX
IMPORTS K-LATEX+PL-ID+PL-INT
SYNTAX Exp ::= Exp * Exp [strict]
          DeclId
          Id
          Int
          Exp > Exp [strict]
          Exp ++
          Exp == Exp [strict]
          Exp != Exp [strict]
          Exp < Exp [strict]
          Exp <= Exp [strict]
          Exp % Exp [strict]
          ! Exp
          Exp && Exp
          Exp ? Exp : Exp
          Exp [ ] Exp
          printf("hd", Exp) [strict]
          scanf("hd", &Exp) [strict]
          scanf("hd", Exp) [strict]
          NULL
          PointerId
          (int*)malloc( Exp *sizeof(int)) [strict]
          free( Exp ) [strict]
          * Exp [strict]
          Exp [ Exp ]
          Exp = Exp [strict(2)]
          Id ( List(Exp) ) [strict(2)]
          Id ( )
          random()
          srandom( Exp ) [strict]
SYNTAX Stmt ::= Exp ; [strict]
          { }
          { StmtList }
          if( Exp ) Stmt
          if( Exp ) Stmt else Stmt [strict(1)]
          while( Exp ) Stmt
          return Exp ; [strict]
          DeclId List(DeclId) { StmtList }
          #include< SimtList >
SYNTAX SimtList ::= SimtList SimtList
          SimtList
SYNTAX Pgm ::= SimtList
SYNTAX Id ::= main
SYNTAX PointerId ::= * PointerId (ditto)
          Id
SYNTAX DeclId ::= int Exp
          void PointerId
SYNTAX SimtList ::= stdio.h
          stdlib.h
SYNTAX List(Bottom) ::= List(Bottom) , List(Bottom) [assoc hybrid id: ( ) strict]
          ( )
          Bottom
SYNTAX List(PointerId) ::= List(PointerId) , List(PointerId) (ditto)
          List(Bottom)
          PointerId
SYNTAX List(DeclId) ::= List(DeclId) , List(DeclId) (ditto)
          DeclId
          List(Bottom)
SYNTAX List(Exp) ::= List(Exp) , List(Exp) (ditto)
          Exp
          List(DeclId)
          List(PointerId)
END MODULE

MODULE KERNELC-DESUGARED-SYNTAX
IMPORTS K-LATEX
IMPORTS KERNELC-SYNTAX
MACRO ! E = E ? 0 : 1
MACRO E1 && E2 = E1 ? E2 : 0
MACRO E1 [ ] E2 = E1 ? 1 : E2
MACRO if( E ) St = if( E ) St else { }
MACRO NULL = 0
MACRO I ( ) = I ( ( ) )
MACRO int * PointerId = int PointerId
MACRO #include< Simt > = Simt
MACRO E1 [ E2 ] = * E1 * E2
MACRO scanf("hd", & * E) = scanf("hd", E)
MACRO int * PointerId = E = int PointerId = E
MACRO int X = E ; = int X ; X = E ;
MACRO stdio.h = { }
MACRO stdlib.h = { }
END MODULE

```

MODULE KERNELC-SEMANTICS
IMPORTS K-SHARED
IMPORTS K-KERNELC-DESUGARED-SYNTAX+PL-CONVERSION+PL-RANDOM
CONFIGURATION:



RULE $I_1 + I_2 \rightarrow I_1 +_{int} I_2$

RULE $I_1 - I_2 \rightarrow I_1 -_{int} I_2$

RULE $I_1 \% I_2 \rightarrow I_1 \%_{int} I_2$ when $I_2 != 0$

RULE $I_1 < I_2 \rightarrow Bool$ ($I_1 <_{int} I_2$)

RULE $I_1 < I_2 \rightarrow Bool2int$ ($I_1 <_{int} I_2$)

RULE $I_1 == I_2 \rightarrow Bool2int$ ($I_1 ==_{int} I_2$)

RULE $I_1 != I_2 \rightarrow Bool2int$ ($I_1 !=_{int} I_2$)

RULE $? ; _ \rightarrow if(?) _ else _$

RULE $if(I) - else St \rightarrow St$

RULE $if(I) St else - \rightarrow St$

RULE $while(E) \{ St \} \rightarrow St$

RULE $printf("hd", N) ;$

RULE $scanf("hd", N) ;$

RULE $scanf("hd", & X) ;$

RULE $V ; \rightarrow$

RULE $\{ St \} \rightarrow St$

RULE $\{ \} \rightarrow$

RULE $St St \rightarrow St \sim St$

RULE $int X Xi \{ St \} ;$

RULE $void X Xi \{ St \} ;$

RULE $int X ;$

RULE $int X = E ;$

RULE $int X = E ;$

RULE $int X = E ;$

RULE $int X = E ;$

RULE $int X = E ;$

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RULE $int X = E ;$

RULE $int X = E ;$

RULE $int X = E ;$

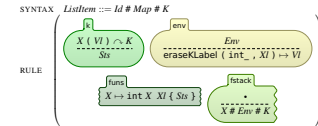
RULE $int X = E ;$

RULE $int X = E ;$

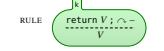
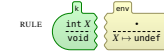
RULE $int X = E ;$

RULE $int X = E ;$

RULE $int X = E ;$



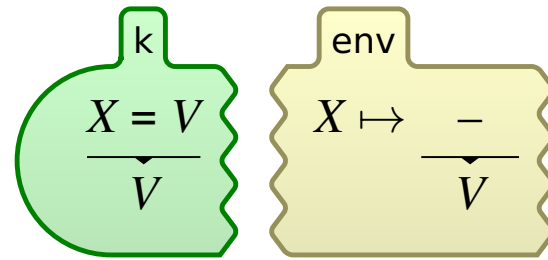
CONTEXT: int = 0



RULE $random() ;$

RULE $random() ;$

Semantic rules given contextually

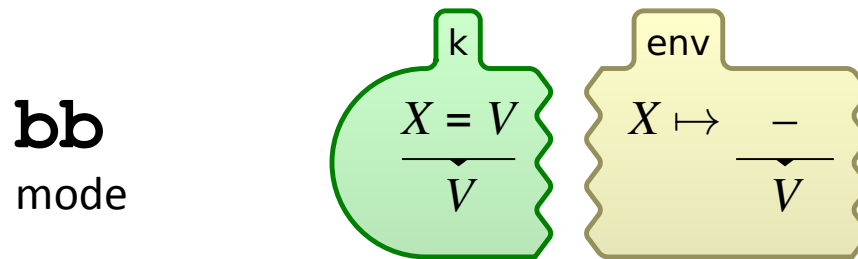


$\langle k \rangle X = V \Rightarrow V \dots \langle /k \rangle$

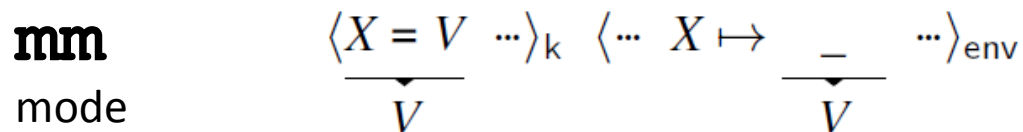
$\langle env \rangle \dots X \mapsto (_ \Rightarrow V) \dots \langle /env \rangle$

Don't Like Bubbles?

- The KernelC definition above was generated by the K tool using the bubble mode (**bb**)



- K tool also provides a mathematical mode (**mm**), which may be preferred in formal writing



What is K, after all ... ?

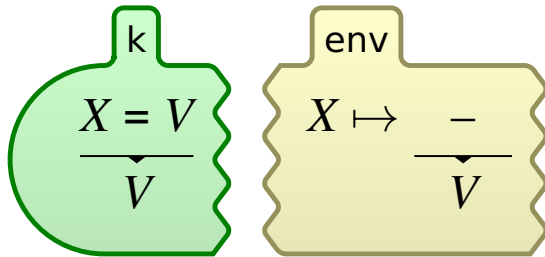
Except for its true concurrency semantics, based on graph rewriting, K is a technique and notation to define languages as rewrite systems with rules

$$l \Rightarrow r \text{ if } b \quad (b \text{ is a side condition})$$

... and so are reduction semantics with evaluation contexts, (chemical) abstract machines, etc.

Translating K into rules $l \Rightarrow r$ if b

SYNTAX $Exp ::= Exp = Exp$ [strict(2)]



kompile into:

RULE $E_1 = E_2 \Rightarrow E_2 \curvearrowright E_1 = \square$ if $E_2 \notin Val$

RULE $E_2 \curvearrowright E_1 = \square \Rightarrow E_1 = E_2$ if $E_2 \in Val$

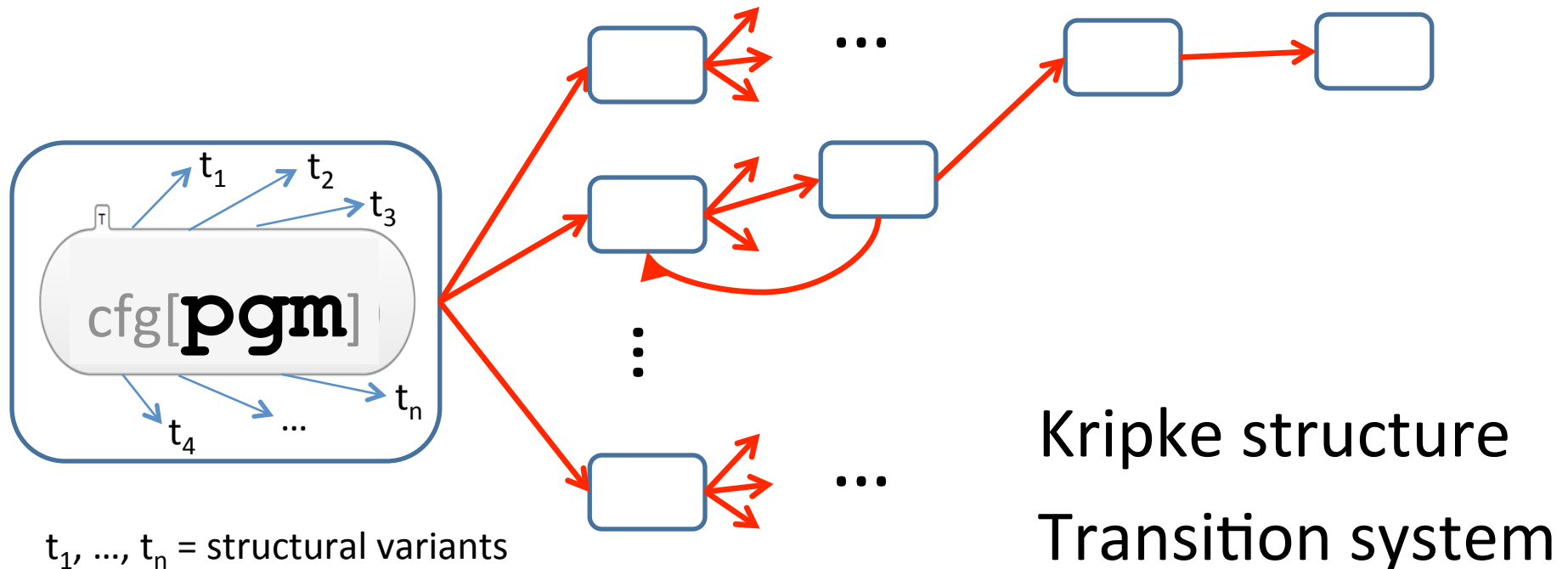
RULE $k(X = V \curvearrowright K) \text{ env}(\rho_1, X \mapsto V', \rho_2)$
 $\Rightarrow k(V \curvearrowright K) \text{ env}(\rho_1, X \mapsto V, \rho_2)$

What is the K Semantics of a Program?

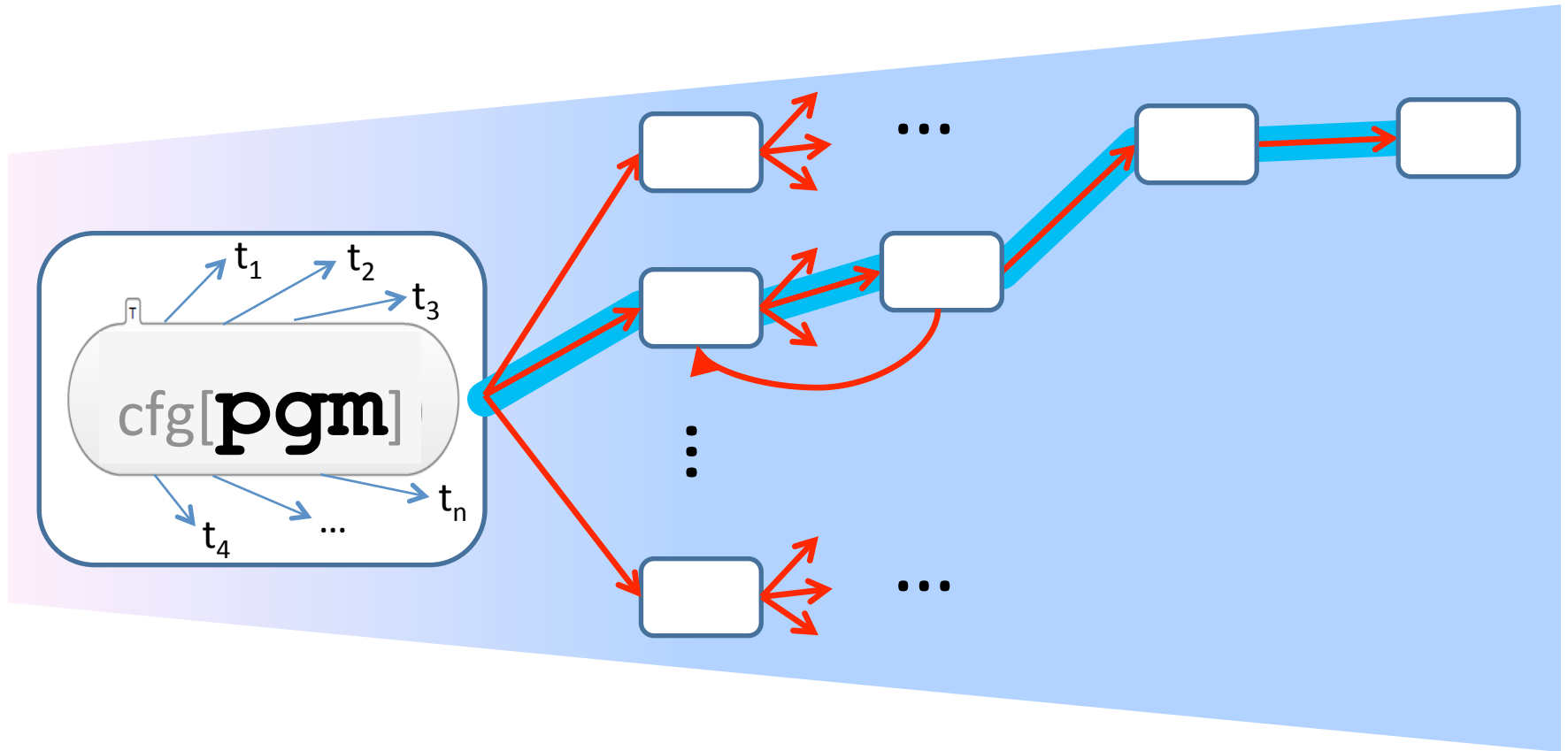
Two types of K rules:

Structural: rearrange configuration, unobservable

Computational: count as computational steps



What does the K Tool Offer?



Efficient and interactive execution (interpreters)
State-space exploration (search and model-checking)

K Semantics are Useful

- Executable, help language designers
- Make teaching PL concepts hands-on and fun
- Currently compiled into
 - Maude, for execution, debugging, model checking
 - Latex, for human inspection and understanding
- Plans to be compiled to
 - OCAML, for fast execution
 - COQ, for meta-property verification

Medium-Size K Definition

See the

- [dynamic semantics](#) or the
- [type checker](#)

of SIMPLE

K Scales

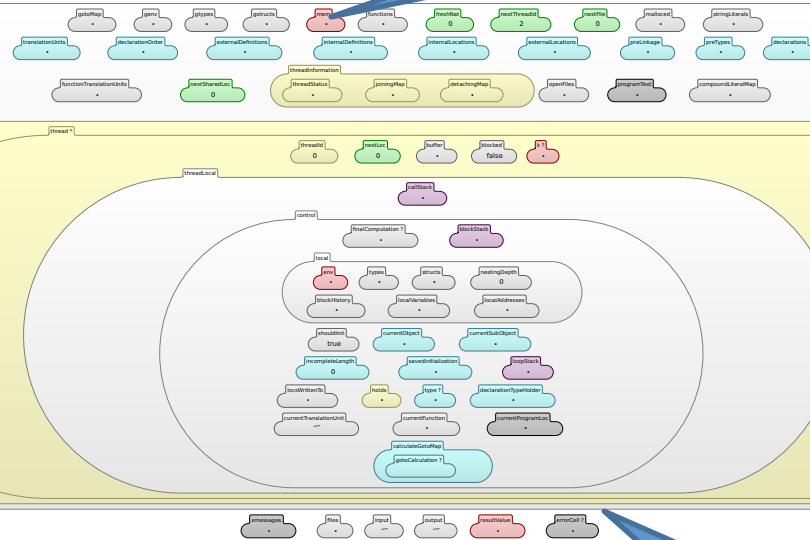
Besides smaller and paradigmatic teaching languages, several larger languages were defined

- Scheme : by Pat Meredith
- Java 1.4 : by Feng Chen
- Verilog : by Pat Meredith and Mike Katelman
- C : by Chucky Ellison

etc.

The K Configuration of C

Heap



75 Cells!

Statistics for the C definition

- Total number of rules: **~1200**
- Has been tested on thousands of C programs (several benchmarks, including the gcc torture test, code from the obfuscated C competition, etc.)
 - Passed **99.2%** so far!
 - GCC 4.1.2 passes 99%, ICC 99.4%, Clang 98.3 (no opt.)
- *The most complete formal C semantics*
- Took more than 18 months to define ...
 - Wouldn't it be uneconomical to redefine it in each tool?

Matching Logic Verification

=

Rewriting (language semantics)

+

[FOL] (configuration reasoning)

+

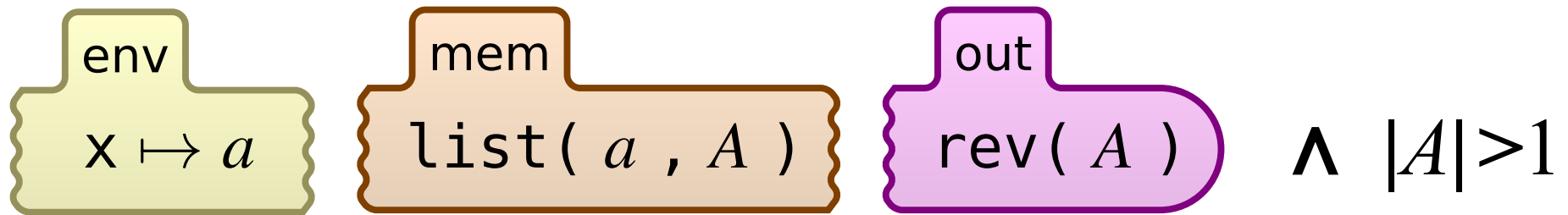
Proof Rules (behavior reasoning)

Matching Logic

- A logic for reasoning about configurations
- Formulae
 - [FOL] over configurations, called **patterns**
 - Configurations are allowed to contain variables
- Models
 - Ground configurations
- Satisfaction
 - **Matching** for configurations, plus [FOL] for the rest

Examples of Patterns

- x points to sequence A with $|A| > 1$, and the reversed sequence $\text{rev}(A)$ has been output



- **untrusted()** can only be called from **trusted()**



More Formally: Configurations

- For concreteness, assume configurations having the following syntax:

$$\langle \langle \dots \rangle_k \langle \dots \rangle_{\text{env}} \langle \dots \rangle_{\text{heap}} \langle \dots \rangle_{\text{in}} \langle \dots \rangle_{\text{out}} \dots \rangle_{\text{cfg}}$$

(matching logic works with any configurations)

- Examples of concrete (ground) configurations:

$$\begin{aligned} &\langle \langle x=*y; y=x; \dots \rangle_k \langle x \mapsto 7, y \mapsto 3, \dots \rangle_{\text{env}} \langle 3 \mapsto 5 \rangle_{\text{heap}} \dots \rangle_{\text{cfg}} \\ &\langle \langle x \mapsto 3 \rangle_{\text{env}} \langle 3 \mapsto 5, 2 \mapsto 7 \rangle_{\text{heap}} \langle 1, 2, 3, \dots \rangle_{\text{in}} \langle \dots, 7, 8, 9 \rangle_{\text{out}} \dots \rangle_{\text{cfg}} \end{aligned}$$

More Formally: Patterns

- Concrete configurations are already patterns, but very simple ones, ground
- Example of more complex pattern

$$\exists c:Cells, e:Env, p:Nat, i:Int, \sigma:Heap$$
$$\underline{\langle \langle x \mapsto p, e \rangle_{env} \langle p \mapsto i, \sigma \rangle_{heap} c \rangle_{cfg} \wedge i > 0 \wedge p \neq i}$$

- Thus, patterns generalize both terms and [FOL]
- Models: concrete configurations + valuations
- Satisfaction: matching for patterns, [FOL] for rest

More Formally: Reasoning

- We can now prove (using FOL reasoning) properties about configurations, such as

$$\models \forall c:Cell, e:Env, p:Nat \\ \langle \langle \mathbf{x} \mapsto p, e \rangle_{env} \langle p \mapsto 9 \rangle_{heap} c \rangle_{cfg} \wedge p > 10 \\ \rightarrow \exists i:Int, \sigma:Heap \\ \langle \langle \mathbf{x} \mapsto p, e \rangle_{env} \langle p \mapsto i, \sigma \rangle_{heap} c \rangle_{cfg} \wedge i > 0 \wedge p \neq i$$

Matching Logic vs. Separation Logic

- Matching logic achieves separation through matching at the structural (term) level, not through special logical connectives (*)
- Matching logic realizes separation at all levels of the configuration, not only in the heap
 - the heap was only 1 out of the 75 cells in C's def.
- Matching logic can stay within FOL, while separation logic needs to extend FOL
 - Thus, we can use the existing SMT provers, etc.

Matching Logic Verification

- Hoare style - **not recommended**

$$\{\pi_{\text{pre}}\} \text{ code } \{\pi_{\text{post}}\}$$

– Need to redefine the PL semantics – **impractical**

- Rewriting (or K) style – **recommended**

$$\textit{left}[\text{code}] \rightarrow \textit{right}$$

– Can reuse existing operational semantics – **good**

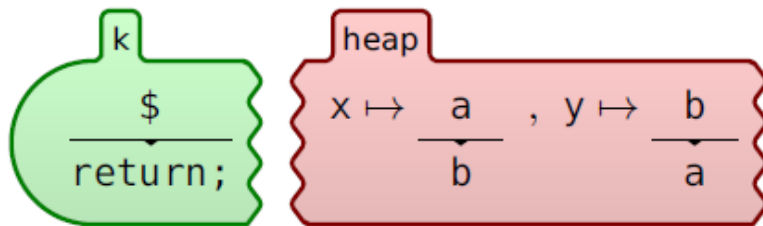
Example – Swapping Values

$\$$ {


```

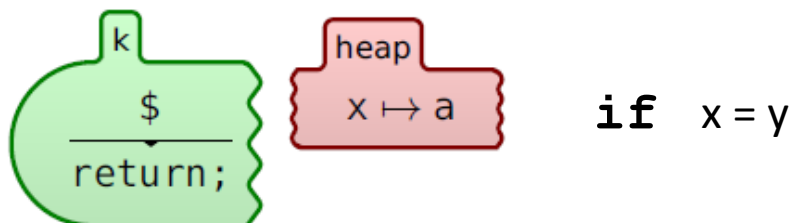
void swap(int *x, int *y)
{
    int t;
    t=*x;
    *x=*y;
    *y=t;
}
        
```

- What is the K semantics of the swap function?
- Let $\$$ be its body



```

rule <k> $ => return; ...</k>
    <heap>...
        x|->(a=>b) ,
        y|->(b=>a)
    ...</heap>
    
```



```

rule <k> $ => return; ...</k>
    <heap>... x|-> a ...</heap>
    if x = y
    
```

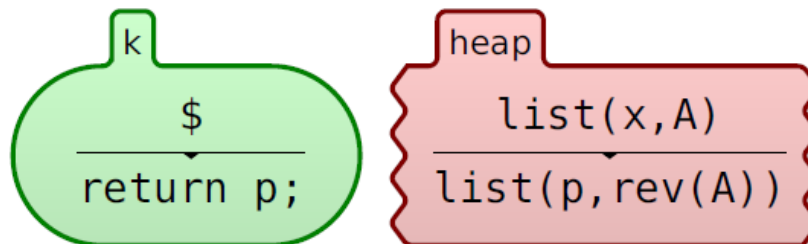
Example – Reversing a list

```

struct listNode* reverse(struct listNode *x)
{
    struct listNode *p;
    struct listNode *y;
    p = 0 ;
    while(x) {
        y = x->next;
        x->next = p;
        p = x;
        x = y;
    }
    return p;
}

```

- What is the K semantics of the reverse function?
- Let \$ be its body



rule $\langle k \rangle \ \$ \Rightarrow \text{return } p; \ \langle /k \rangle$
 $\langle \text{heap} \rangle \dots \text{list}(x, A) \Rightarrow \text{list}(p, \text{rev}(A)) \ \dots \langle / \text{heap} \rangle$

Partial Correctness

- We have two rewrite relations on configurations
 - given by the language K semantics; **safe**
 - given by specifications; **unsafe**, has to be proved
- Idea (simplified for deterministic languages):
 - Pick **left** → **right**. Show that always **left** → (**→** ∪ →)* **right** modulo matching logic reasoning (between rewrite steps)
- Theorem (soundness):
 - If **left** → **right** and “**config** matches **left**” such that **config** has a normal form for →, then “**nf(config)** matches **right**”

More Formally:

Matching Logic Rewriting

- Matching logic rewrite rules are rewrite rules over matching logic formulae: $\varphi \Rightarrow \varphi'$
- Since patterns generalize terms, matching logic rewriting captures term rewriting
- Moreover, deals naturally with side conditions: rewrite rules of the form

$$l \Rightarrow r \text{ if } b$$

are captured as matching logic rules of the form

$$l \wedge b \Rightarrow r$$

More Formally: Proof System I

- Rules of operational nature

Reflexivity

$$\frac{\cdot}{\mathcal{A} \vdash \varphi \Rightarrow \varphi}$$

Axiom

$$\frac{\varphi \Rightarrow \varphi' \in \mathcal{A}}{\mathcal{A} \vdash \varphi \Rightarrow \varphi'}$$

More Formally: Proof System II

Substitution

$$\frac{\mathcal{A} \vdash \varphi \Rightarrow \varphi' \quad \theta : Var \rightarrow \mathcal{T}_{\Sigma}(Var)}{\mathcal{A} \vdash \theta(\varphi) \Rightarrow \theta(\varphi')}$$

Transitivity

$$\frac{\mathcal{A} \vdash \varphi_1 \Rightarrow \varphi_2 \quad \mathcal{A} \vdash \varphi_2 \Rightarrow \varphi_3}{\mathcal{A} \vdash \varphi_1 \Rightarrow \varphi_3}$$

More Formally: Proof System III

- Rules of deductive nature

Case analysis

$$\frac{\mathcal{A} \vdash \varphi_1 \Rightarrow \varphi \quad \mathcal{A} \vdash \varphi_2 \Rightarrow \varphi}{\mathcal{A} \vdash \varphi_1 \vee \varphi_2 \Rightarrow \varphi}$$

Logic framing

$$\frac{\mathcal{A} \vdash \varphi \Rightarrow \varphi' \quad \psi \text{ is a FOL}_= \text{ formula}}{\mathcal{A} \vdash \varphi \wedge \psi \Rightarrow \varphi' \wedge \psi}$$

More Formally: Proof System IV

Consequence

$$\frac{\models \varphi_1 \rightarrow \varphi'_1 \quad \mathcal{A} \vdash \varphi'_1 \Rightarrow \varphi'_2 \quad \models \varphi'_2 \rightarrow \varphi_2}{\mathcal{A} \vdash \varphi_1 \Rightarrow \varphi_2}$$

Abstraction

$$\frac{\mathcal{A} \vdash \varphi \Rightarrow \varphi' \quad X \cap \text{FreeVars}(\varphi') = \emptyset}{\mathcal{A} \vdash \exists X \varphi \Rightarrow \varphi'}$$

More Formally: Proof System V

- Main proof rule of matching logic rewriting

Circularity

$$\frac{\mathcal{A} \vdash \varphi \Rightarrow^+ \varphi'' \quad \mathcal{A} \cup \{\varphi \Rightarrow \varphi'\} \vdash \varphi'' \Rightarrow \varphi'}{\mathcal{A} \vdash \varphi \Rightarrow \varphi'}$$

Fact

- Matching logic generalizes both operational semantics and axiomatic semantics
 - Operational semantics by means of capturing term rewriting as discussed above
 - Axiomatic semantics by noticing that Hoare triples are particular pattern rewrites:

$$\text{HL2ML}(\{\psi\} s \{\psi'\}) =$$

$$\langle s, \sigma_Z \rangle \wedge \sigma_Z(\psi) \Rightarrow \exists Z(\langle \text{skip}, \sigma_Z \rangle \wedge \sigma_Z(\psi'))$$

Theorems

- Any operational behavior can also be derived using matching logic reasoning
- For any Hoare triple $\{\psi\} s \{\psi'\}$ derived with axiomatic semantics, the corresponding matching logic rule $\text{HL2ML}(\{\psi\} s \{\psi'\})$ can be derived with the matching logic proof system
 - Proof is constructive, not existential
- Partial correctness
 - Holds for ALL languages

MatchC

- A Matching Logic Verifier for (a fragment of) C
- Uses K semantics of C fragment *unchanged*
- Has verified a series of challenging programs
 - Undefinedness, typical Hoare-like programs, heap programs (lists, trees, stacks, queues, graphs), sortings, AVL trees, Schorr-Waite graph marking

Conclusions

- K (semantics) and Matching Logic (verification)
- Formal semantics is useful and practical!
- One can use an executable semantics of a language *as is* also for program verification
 - As opposed to redefining it as a Hoare logic
- Giving a formal semantics is not necessarily painful, it can be fun if one uses the right tools